

New constitutive relation for eddy viscosity models

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Abstract: A new turbulent constitutive relation was directly derived from Boussinesq's hypothesis and mixing length theory, and then implemented in the standard $k-\epsilon$ model. The performance of this constitutive relation was validated in zero pressure gradient flat-plate boundary layer flow, fully-developed turbulent channel flow and separated flow in a plane asymmetric diffuser. The investigation demonstrated that, this new constitutive relation gave very accurate results in the former two basic cases and provided significant improvement in prediction of separated and reattachment points in the plane asymmetric diffuser. Separation and reattachment points at $x/H=7.5$ and 29 were calculated accurately in comparison to experimental results, and the static pressure coefficient of 0.82 was very close to large eddy simulation calculation. These results are very encouraging but further verification and extensive application of the new constitutive relation to other two-equation eddy viscosity model are needed.

Key words: constitutive relation; turbulence model; flat-plate flow; channel flow; separated flow

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Reynolds averaged turbulence (RANS) model, particularly standard two-equation models continue to constitute the dominant approach for representing the effects of turbulence in prediction procedures for industrial flows. However, linear eddy viscosity turbulence models (EVM) can not give accurate prediction in flows with strong effects of streamline curvature, adverse pressure gradients, system rotation, separation or reattachment. Even in relatively simple shear flow, these models which are based on Boussinesq's hypothesis are unable to accurately predict normal Reynolds stress. On the other hand, Reynolds stress turbulence model are based on the Reynolds stress transport equations which automatically account for the convection and diffusion of Reynolds stress and consequently adjust themselves in turbulent flows with streamline curvature or system rotation. In the

last a few decades Reynolds-stress model has been demonstrated to be superior to two-equation models especially for complicated flows. In parallel with such efforts there has been a remarkable renewed interest in development of a more general constitute relation of turbulent flows, e. g., nonlinear eddy viscosity models (NLEVM).

The oldest and simplest proposal for modeling the turbulent stress is Boussinesq's isotropic hypothesis which is at the heart of eddy viscosity models. But it is well known that this kind eddy viscosity model has two major deficiencies: first it is purely dissipative hence it can not account for Reynolds stress relaxation effects; second it is oblivious to the presence of rotational strains. In order to overcoming these deficiencies, many researchers focus on development of nonlinear or anisotropic generalizations of eddy viscosity models.

The earliest work on the development of turbulence models with a nonlinear stress-rate of strain to be empirical in nature, relying on analogies with laminar non-Newtonian flows by Rivlin^[1]. Lumley^[2] found that turbulent constitutive relations may exist only under the situation in which the length and time scales of turbulence are much smaller than those in the mean flow field so that the effect of initial and boundary conditions on the turbulence can neglect. However, in most practical situations, this condition is never satisfied hence so called general turbulent constitutive relations do not exist. Pope^[3] developed a nonlinear Reynolds stress model using general weak equilibrium assumption and presented a methodology for obtaining an explicit relation for the Reynolds stress based on the modelled Reynolds stress equations proposed by Launder, Shih, et al.^[4-5] derive a general constitutive relation for the Reynolds stress using invariant theory and generalized Cayley-Hamilton formulations. Their model is based on the constraints from rapid distortion theory and realizability. Yoshizawa^[6] derived a two-equation model by means of two-scale direct interaction approximation method. Rubinstein and Barton^[7] proposed a nonlinear model of anisotropic turbulence in which the Reynolds stresses are quadratic functions of the mean velocity gradients with the renormalization group method. Many other nonlinear eddy viscosity models have been proposed by Speziale, Craft, Apsley and Huang, et al.^[8-11]. These nonlinear models mentioned above became more and more complicated but none achieves great width of applicability.

It will be helpful to mention the scale-adaptive simulation (SAS) method proposed by Menter, et al.^[12] He started from the k - kL model developed by Rotta^[13] earlier and retained the second velocity derivative term neglected by Rotta. Finally he added a second velocity derivative term in the ω equation, which is sensitive to resolved fluctuations. Menter, et al.^[14] argued that the inclusion of the second velocity derivative allows shear stress transport (SST) k - ω based SAS

model to resolve turbulent structures without dissipation them as classical RANS models would.

In the present work, a new kind of relation between Reynolds stress and first and second spatial derivatives of velocity is presented. The motivation for this work is the general need for improvements in the prediction of complicated flows based on the two-equation modelling level, especially k - ϵ model at present.

1 The new constitutive relation

The start point of Boussinesq's hypothesis is an analogy to kinetic theory because the similarity between stress $\overline{u''v''}$ produced by random molecular fluctuations u'' and v'' and stress $\overline{u'v'}$ produced by turbulent fluctuations u' and v' under Reynolds average. The simple situation where molecules have a mean velocity of the form $(U(y), 0, 0)$ is often taken as an illustrative example^[15], as Fig. 1(a) shown, the net shear stress is

$$t_{xy} = \overline{u''v''} = \frac{1}{4} \nu_{th} [U(y + l_{mfp}) - U(y - l_{mfp})] \approx \frac{1}{2} \nu_{th} l_{mfp} dU/dy \quad (1)$$

where the gradient is in fact a one order approximation of the finite difference quantity $U(y + l_{mfp}) - U(y - l_{mfp})$, t_{xy} is molecular shear stress, ν_{th} is the thermal velocity, l_{mfp} is the mean free path of molecules. Since this distance is very small compared to the scale of variation of U , the above approximation is clearly justified. Usually $\nu = 1/2 \nu_{th} l_{mfp}$ is set, where ν is the kinematic viscosity.

For turbulent flows the same construction is implicitly assumed. Prandtl introduced the mixing-length concept. He made an assumption that each lump of fluid that is subjected to the turbulent motion may be considered as an individual unit. He further assumed that is a shear flow such as depicted in Fig. 1(b), the lumps retain their x -direction momentum for a distance in the y direction, l_{mix} , which was called the mixing length. A equation for turbulent viscosity (ν_t) is similar to Eq. (1) using an analogy with kine-

matic viscosity

$$\tau_{xy} = 1/2 v_{\text{mix}} l_{\text{mix}} \frac{dU}{dy} = \nu_t \frac{dU}{dy} \quad (2)$$

As pointed out by Wilcox, there are two implicit assumptions in the analysis of kinematic viscosity that require justification. First the Knudsen number must be far small than unit, $Kn = l_{\text{mfp}}/L \ll 1$, where L is the characteristic length of mean flow. Second the thermal motion velocity u'' remained Maxwellian even in the presence of shear which require that the average time between molecular collisions is far small than the characteristic time scale of the mean flow, $l_{\text{mfp}}/v_{\text{th}} \ll 1/|dU/dy|$. Besides above two assumptions, there is another assumption must be realized: each molecule possesses a free path l , which is a random variable whose distribution is peaked around a mean value l_{mfp} , the mean free path. In kinetic theory, all these three assumptions are well satisfied but in the mixing length theory of turbulent flow are not. This means that: (1) the separation of scales does not hold in general for turbulent flows and one order approximation in Eq. (1) is suspect because the first assumption is not satisfied. (2) The momentum of the fluid lump will not be preserved and consequently it is not a transferable quantity because the second assumption is not satisfied. (3) The mixing length is not a fluid property and is unhomogeneous in turbulent flows because the third assumption is not satisfied. Therefore the mixing length theory had been better reconsider.

Accounting for higher order approximation and unhomogeneous mixing length in turbulent flows as shown in Fig. 1 (b), turbulent shear stress is prescribed as:

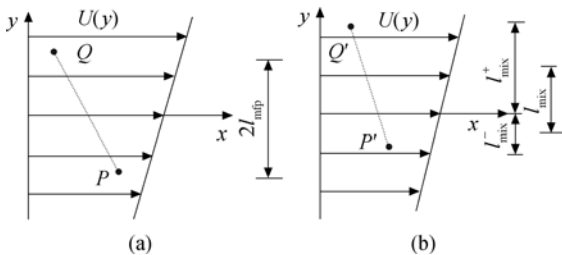


Fig. 1 Schematic of shear flow

$$\begin{aligned} \tau_{xy} &= 1/4 v_{\text{mix}} [U(y + l_{\text{mix}}^+) - U(y - l_{\text{mix}}^-)] \approx \\ &1/4 v_{\text{mix}} (l_{\text{mix}}^+ + l_{\text{mix}}^-) dU/dy + \\ &1/8 v_{\text{mix}} [(l_{\text{mix}}^+)^2 - (l_{\text{mix}}^-)^2] d^2U/dy^2 \end{aligned} \quad (3)$$

In homogeneous flow, the second order term can be neglected but in most situation turbulent flows are unhomogeneous. If ν_t was defined as $\nu_t = 1/2 v_{\text{mix}} l_{\text{mix}} = 1/4 v_{\text{mix}} (l_{\text{mix}}^+ + l_{\text{mix}}^-)$ and $l_{\text{mix}}^+ - l_{\text{mix}}^- \propto dl_{\text{mix}}/dy l_{\text{mix}}$, then implement Eq. (3) to standard k - ϵ model proposed by Launder and Spalding^[16] in which $l_{\text{mix}} = C_{\mu}^{3/4} k^{3/2} / \epsilon$,

$$\begin{aligned} \tau_{xy} &= C_{\mu} \frac{k^2}{\epsilon} \cdot \frac{dU}{dy} + \\ &C_{\mu^2} C_{\mu}^{5/2} \frac{k^4}{\epsilon^4} \left(\frac{3}{2} \epsilon \frac{dk}{dy} - k \frac{d\epsilon}{dy} \right) \frac{d^2U}{dy^2} \end{aligned} \quad (4)$$

Where k is turbulent kinetic energy, ϵ is turbulent dissipation rate.

It is easy to extend this equation to three-dimensions

$$\begin{aligned} \tau_{ij} &= -\frac{2}{3} k \delta_{ij} + C_{\mu} \frac{k^2}{\epsilon} S_{ij} + \\ &C_{\mu^2} C_{\mu}^{5/2} \frac{k^4}{\epsilon^4} \left(\frac{3}{2} \epsilon \frac{dk}{dy} - k \frac{d\epsilon}{dy} \right) \frac{d}{dx_k} \left(S_{ij} - \frac{1}{3} S_{mm} \delta_{ij} \right) \end{aligned} \quad (5)$$

where δ_{ij} is Kronecker delta, S_{ij} is rate of strain tensor, C_{μ^2} is a coefficient which is set to unit at present.

In two-dimensional separated flow, the conventional constitutive relation of eddy viscosity will give zero turbulence viscosity at separated point because the first velocity derivative is zero. This is untrue physically. In the present work, the second velocity derivative directly from the Boussinesq's hypothesis and mixing length concept were introduced. The new constitutive relation contains more information of the mean flow field. Hence it will be sensible to introduce second velocity derivative into turbulence modeling, and better prediction of separated flows is expected.

Here the equations and the models were summarized, which will be used for application in the next section. For incompressible flows, the governing equations are

$$\frac{\partial U_i}{\partial x_i} = 0 \quad (6)$$

$$\frac{\partial U_i}{\partial t} + \frac{\partial (U_i U_j)}{\partial x_j} = \frac{1}{\rho} \cdot \frac{\partial p}{\partial x_i} + \frac{\partial (\nu S_{ij})}{\partial x_j} + \frac{\partial (-u'_i u'_j)}{\partial x_j} \quad (7)$$

where p is averaged pressure, the Reynolds stresses will be modeled by Eq. (5)

$$-\overline{u'_i u'_j} = \tau_{ij} = -\frac{2}{3} k \delta_{ij} + C_\mu \frac{k^2}{\epsilon} S_{ij} + C_{\mu 2} C_\mu^{5/2} \frac{k^4}{\epsilon^4} \left[\frac{3}{2} \epsilon \frac{dk}{dy} - k \frac{d\epsilon}{dy} \right] \frac{d}{dx_k} \left[S_{ij} - \frac{1}{3} S_{mn} \delta_{ij} \right]$$

Where $C_\mu = 0.09$, $C_{\mu 2} = 1.0$.

In the present study, the standard k - ϵ model is

$$\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = \tau_{ij} \frac{\partial U_i}{\partial x_j} - \epsilon + \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] \quad (8)$$

$$\frac{\partial \epsilon}{\partial t} + U_j \frac{\partial \epsilon}{\partial x_j} = C_{\epsilon 1} \frac{\epsilon}{k} \tau_{ij} \frac{\partial U_i}{\partial x_j} - C_{\epsilon 2} \frac{\epsilon^2}{k} + \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_\epsilon} \right) \frac{\partial \epsilon}{\partial x_j} \right] \quad (9)$$

where

$$\nu_t = C_\mu k^2 / \epsilon \quad (10)$$

The coefficients $C_{\epsilon 1}$, $C_{\epsilon 2}$, σ_k and σ_ϵ assume their standard value: $C_{\epsilon 1} = 1.44$, $C_{\epsilon 2} = 1.92$, $\sigma_k = 1.0$, $\sigma_\epsilon = 1.3$.

2 Applications

2.1 Numerical solution methods

Computations are performed with the open source CFD code OpenFOAM^[17], which uses the finite volume method to solve systems of partial differential equations discretized on any three-dimensional unstructured mesh of polygonal cells. All solvers developed within OpenFOAM are innately three-dimensional but can be applied to two-dimensional problems by using a particular condition on boundaries lying in the plane of the third direction. A few classical turbulence models, for example, the k - ϵ , k - ω and Spalart-Allmaras (S-A) models, are already implemented in OpenFOAM. The new model was implemented into the k - ϵ model.

Preconditioned (bi-)conjugate gradient matrix solver methods are used to solve the discretised matrix equations. Variable storage is co-located and cell-centred, with the Rhie-Chow algo-

rithm used to prevent checkerboard oscillations in the pressure field. The advective terms are discretized using second-order total variation diminishing (TVD) limited linear scheme, whilst the diffusive terms are discretized using a second-order central scheme. The pressure/velocity linkage is dealt with using the semi-implicit method for pressure linked equations (SIMPLE) algorithm to iterate the system to a steady-state solution. The Reynolds averaged N-S equations and turbulence model equations are solved using an under-relaxation method which provides a stable way for iterating with large pseudo-time step values, and allows for faster convergence. The pressure under-relaxation factor is 0.3 and a value of 0.7 is used for the other equations. Generally, velocity and turbulent variables are prescribed at the inlet while the Neumann condition is used at the outlet. Pressure is set to zero gradient at the inlet and fixed value at the outlet. On the walls, no-slip condition is used, velocity is zero and enhanced wall-functions for the turbulent variables are adopted. More detailed boundary conditions specified in experiment cases will be described in the next sections.

2.2 Zero pressure gradient flat-plate flow

The proposed constitutive relation is calibrated primarily for boundary layer flow. The zero pressure gradient flat-plate flow was experimentally studied by Wiegardt and Tillman^[18]. Flow past the flat-plate is calculated at a Mach number of 0.2 and $Re = 5$ million based on unit length.

The computational grid used was refined until grid independence was achieved. The near-wall grid sizes were chosen to obtain a y^+ value less than 5. At the inlet, velocity and turbulent properties are provided as experimental data. At the wall, no slip condition for velocity and enhanced wall function for k , ϵ , ν_t . Pressure is set to zero gradient at above two type boundaries. At the outlet, pressure is set to a fixed value and the remaining properties are set to zero gradient. All results are compared to experimental data from Ref. [18].

The skin friction coefficient (C_f) profile plotted in Fig. 2 shows that the new constitutive relation ingrated in standard k - ϵ model is able to capture skin friction coefficient along the wall accurately. In Fig. 3, dimensionless mean velocity (U^+) profile at $x=3.487$ m is compared to experimental data tabulated in Ref. [18], the agreement between the numerical result and experimental data is quite encouraging.

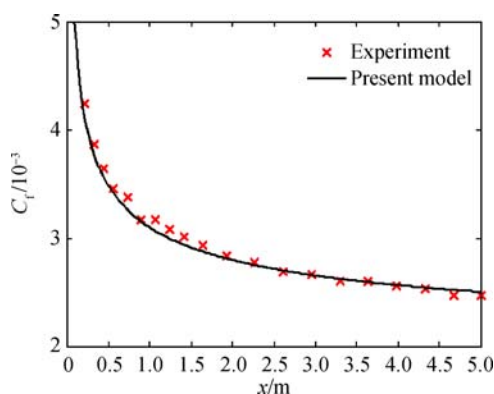


Fig. 2 Skin friction coefficient distribution

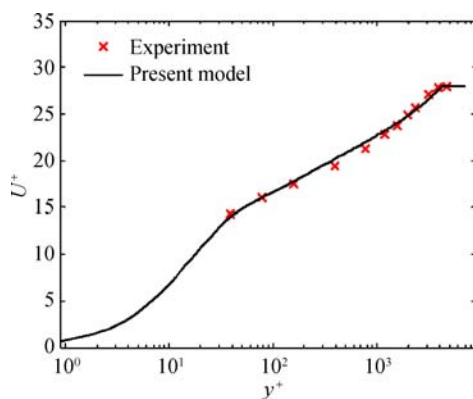


Fig. 3 Mean velocity profile in zero pressure gradient boundary layer at $x=3.487$ m

2.3 Fully-developed turbulent channel flow

The fully-developed turbulent channel flow is one of the standard experiment cases for turbulence models. In fully-developed turbulent channel flow, the x -derivatives in the momentum, temperature, kinetic energy are identically zero, hence the mean velocity field is only dependent on the wall-normal direction y and it is driven by a constant pressure gradient in the streamwise direction. The channel flow is widely

experimentally studied here Wei's experimental data^[19] at $Re=40\,000$ used to verify the present model.

The computational grid used was refined until grid independence was achieved. The near-wall grid sizes were chosen to satisfy $y^+ \leq 5$. At the inlet, velocity and turbulent properties are directed mapped from downstream location which is $10H$ distance from inlet, where H is channel height. At the wall, no slip condition for velocity and enhanced wall function for k , ϵ and ν_t . At the outlet, pressure is set to a fixed value and the remaining properties are set to zero gradient.

Figure 4 shows a comparison of the computed and the experimental velocity profile, the dimensionless turbulent shear stress R^+ is depicted in Fig. 5. Good agreement between the present model and experimental data has been obtained as shown in Fig. 4 and Fig. 5.

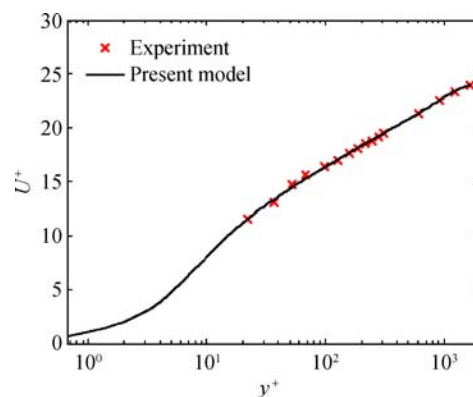


Fig. 4 Mean velocity profile in turbulent channel flow

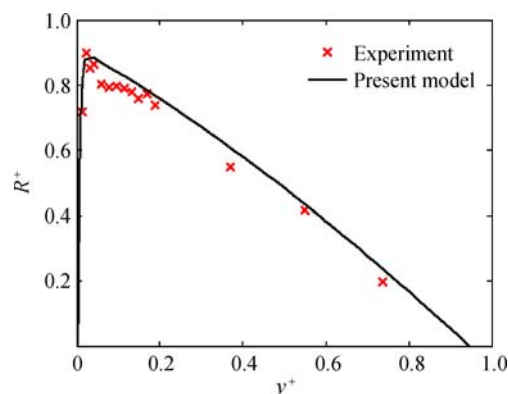


Fig. 5 Shear stress profile in turbulent channel flow

2.4 Plane asymmetric diffuser flow

The third experiment case involves separation from a plane wall and reattachment in the downstream duct of a asymmetric diffuser which is experimentally studied by Buice^[20]. The geometry is shown in Fig. 6. The diffusion section has a length $21H$, where H is the upstream channel height. The asymmetric expansion ratio is 4.7 which giving a diffuser angle of 10 degrees. The Reynolds number based on upstream center line velocity and H is 20 000. The flow was observed to separate from the inclined wall at $x/H=7$ and to reattach at $x/H=29$. The maximum static pressure coefficient is 0.73. The peak skin friction coefficient in the recirculation region is -0.00027 based on the inlet bulk velocity. Mean velocity and turbulent stresses were measured using hot-wire anemometry. Detail measurement technique was reported in Ref. [20].

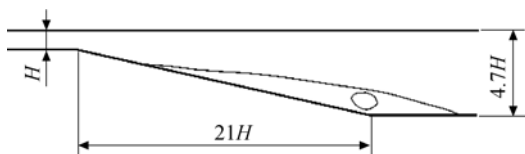


Fig. 6 Plane diffuser geometry

The computation domain extended from $x/H=-40$ to $x/H=135$, contain more than 461×61 points for both standard $k-\epsilon$ model and the present model with enhanced wall functions. The computational grid used was refined until grid independence was achieved. The near-wall grid sizes were chosen to obtain a y^+ value less than 1.7. At the inlet boundary fixed bulk velocity is set since channel flow can develop to fully-developed turbulent flow within such large upstream length. At the wall, no slip condition for velocity and standard wall function for k , ϵ and ν_t . Pressure is set to zero gradient at above two type boundaries. At the outlet, pressure is set to a fixed value and the remaining properties are set to zero gradient. Turbulent properties are calculated from experimental data.

Separation and reattachment points are critical parameters which have often been used to

assess the overall performance of turbulence models. They are determined in this calculation from the points where skin friction coefficient C_f goes to zero. Fig. 7 and Fig. 8 show these skin friction coefficients at inclined wall and straight wall for standard $k-\epsilon$ model and the present model. The recirculation region is from $x/H=7.5$ to $x/H=29$ which is identified by negative C_f . This parameter is very sensitive which was pointed out by Shih, et al.^[5] and Apsley, et al.^[21] This calculation demonstrates the significant improvement in predicting reattachment location by the present model again. And the peak skin friction coefficient in this region is also accurately predicted.

Figure 9 and Fig. 10 show the comparison of computed and measured static pressure coefficients (C_p) along the inclined and straight wall.

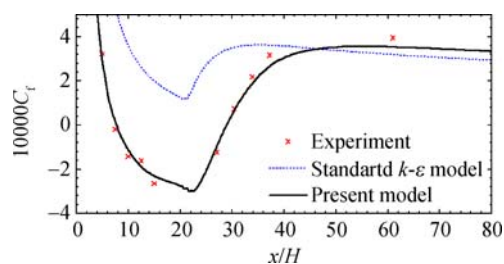


Fig. 7 Skin friction coefficients along inclined wall

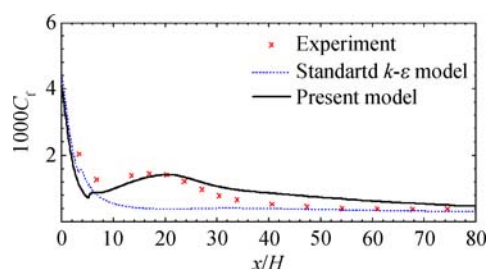


Fig. 8 Skin friction coefficients along straight wall

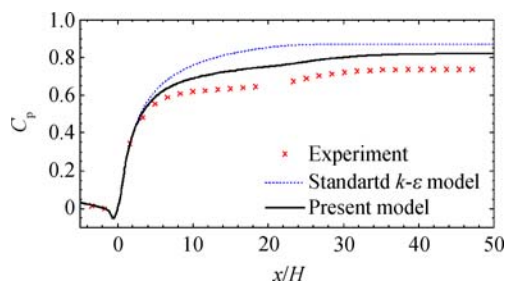


Fig. 9 Static pressure coefficients along inclined wall

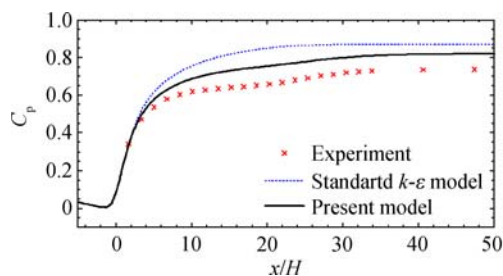


Fig. 10 Static pressure coefficients along straight wall

There is no significant difference between the two walls. Both models overpredict static pressure recovery but the present model get a similar result $C_p = 0.82$ of large eddy simulation conducted by Buice^[22].

The stream mean velocity U profiles are shown in Fig. 11 at ten different cross sections (from left to right, $x/H = -6, 3, 6, 14, 19, 24, 30, 40, 47, 60$), where U_b bulk velocity at inlet. It is evident that the present model produces superior result to standard $k-\epsilon$ model which fail to predict any separation. Fig. 12 to Fig. 14 show the Reynolds stress at various locations. The present model predicts more accurate results than standard $k-\epsilon$ model, especially for the shear stress.

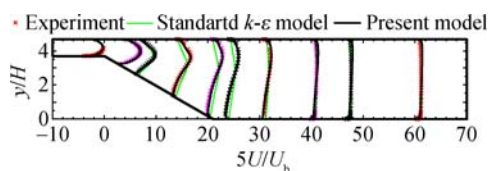
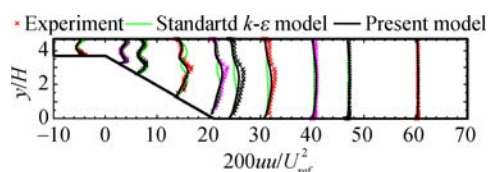
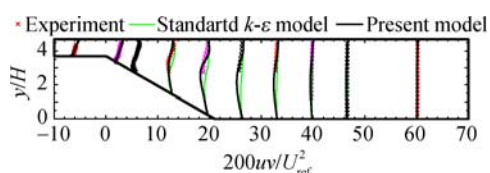
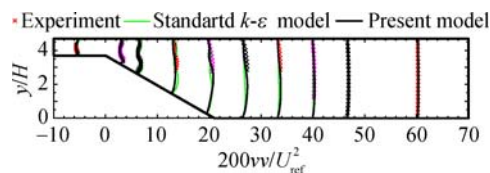


Fig. 11 Mean velocity profile at different position

Fig. 12 Normal stress τ_{xx} profileFig. 13 Shear stress τ_{xy} profileFig. 14 Normal stress τ_{yy} profile

3 Conclusion

A new constitutive relation between Reynolds stresses and rate of strain and second velocity derivative has been proposed. The development of the model is based on reconsideration of Boussinesq's hypothesis and Prandtl's mixing length theory and accounted for the second order approximation in the process of Taylor's expansion in mixing length theory which introduced a second velocity derivative into the constitutive relation. Two benchmark experiment case-zero pressure gradient flat-plate boundary layer flow and fully-developed turbulent channel flow are used to validated this new constitutive relation. The calculation results agree well with experimental data. Comparison over a classical separation flows-plane asymmetric diffuser has shown that the model performs consistently superior to original linear constitutive relation, especially provides significant improvement in prediction of separated and reattachment point ($x/H = 7.5$ and 29) as expected. These findings demonstrate that more information should be extracted from mean velocity field besides the rate of strain. It should mention that the present model implementing the new constitutive relation is as robust and economical as well. This makes the present constitutive relation very attractive for use in engineering applications.

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